REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 074-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	April, 1998	Final, 1 Mar 1995 - 28 Feb 1998

4. TITLE AND SUBTITLE

Research on Gain Scheduling

5. FUNDING NUMBERS

F49620-95-1-0195
PR2304/AS

6. AUTHOR(S)

Wilson J. Rugh

AFRL-SR-BL-TR-98-
0390

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Johns Hopkins University
3400 North Charles Street
Baltimore, MD 21218

None

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

AFOSR/NM
110 Duncan Avenue, Room B115
Bolling AFB, DC 20332

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT

Available to the public.

12b. DISTRIBUTION CODE

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

13. ABSTRACT ***(Maximum 200 Words)***

This final technical report provides a summary and documentation of research accomplishments on methods to improve the performance of gain scheduled controller designs. The main topics involve use of information about scheduling signals, in addition to their current values, to improve performance, the establishment of an optimization framework for the analysis and design of gain scheduled systems, and the development of controller interpolation methods. A list of journal publications is included.

19980430 116

14. SUBJECT TERMS	15. NUMBER OF PAGES
Gain scheduling, Nonlinear optimal Control	6
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

Final Technical Report

Grant Number AFOSR F49620-95-1-0195

Wilson J. Rugh

Department of Electrical and Computer Engineering
Johns Hopkins University
Baltimore, Maryland 21218
rugh@jhu.edu
(410) 516-7004

April, 1998

1. Introduction

This final technical report provides a summary and documentation of research accomplishments on methods to improve the performance of gain scheduled controller designs. The main topics involve use of additional information about scheduling signals, beyond their current values, to improve performance, the establishment of an optimization framework for the analysis and design of gain scheduled systems, and the development of controller interpolation methods.

In Section 2, accomplishments are described and referenced to the journal publications resulting from this effort, as listed in Section 4. Personnel are listed in Section 3, and various presentations and interactions are described in Section 5.

2. Research Accomplishments

In a gain scheduled control system, controller gains are adjusted according to the current value of the so-called scheduling signals. In the case of exogenous scheduling signals, it is sometimes feasible to generate in real time an accurate estimate of the time derivative of the scheduling signals. However it is not reasonable to assume that an exogneous signal generator, the "exosystem," is known.

In [1] we investigate the use of such derivative information in a standard nonlinear control problem. Specifically, the output regulation problem for a nonlinear plant driven by an exogenous signal is addressed. In the absence of the standard exosystem hypothesis, we assume availability of the instantaneous values of the exogenous signal and its first time-derivative for use in the control law. We formulate an approximate output regulation problem and characterize solvability in terms of the existence of a certain invariant manifold. The control law we obtain yields a closed-loop system with output exhibiting ultimate boundedness properties with respect to both the first and the second time-derivatives of the exogenous signal. Performance characteristics are illustrated by application to the linear case and to specific nonlinear examples.

In a similar fashion, output regulation for a discrete-time nonlinear plant driven by an exogenous signal is addressed in [3]. In the absence of the standard exosystem hypothesis, availability of the current and l past values of the exogenous signal for use in the control law is assumed. We formulate an approximate output regulation problem and characterize solvability in terms of the existence of a certain invariant manifold. The control law we obtain yields a closed-loop system with output exhibiting ultimate boundedness properties with respect to the $(l+1)$ difference of the exogenous signal. Performance characteristics are illustrated by application to the linear case and to specific nonlinear examples. A perturbation analysis of robustness properties is also provided.

Although not directly related to the research objectives, we note also the completion of the book [2]. Since gain scheduling relies on linear control methods to address nonlinear control problems, many aspects of basic linear systems and control theory are relevant.

In the course of our efforts to improve the performance of gain scheduled systems, it became clear that an overall framework would be required. Since the local linear designs in a gain scheduling approach often are performed using linear optimal control methods, it seemed natural to attempt to develop a nonlinear optimal control framework for gain scheduling. However because the nonlinear system is operated over a range of operating conditions, corresponding to the value of scheduling signals, the usual optimal-control formulations were inadequate. Thus we turned to an approach now called overtaking optimality that can deal with unbounded

performance indices caused by changing operating conditions. In other words, overtaking optimality is particularly appropriate for formulating problems in optimal gain scheduling because of the presence of the exogenous scheduling signal and its impact on a performance index.

In [6] we consider infinite horizon nonlinear optimal control problems with unbounded performance indices using notions of overtaking optimality. In the framework of optimal control, new sufficient conditions of Caratheodory-Hamilton-Jacobi type for both weak overtaking and overtaking optimality are presented and compared with known results from the calculus of variations setting. Using local structural properties of a related control Hamiltonian, conclusions are drawn on aspects of existence of overtaking and weak overtaking optimal controls, stability of optimal trajectories, and approximation of weak overtaking optimal controls. Both similarities and differences among alternative approaches to overtaking, weak overtaking, and strong optimal control problems are discussed. This formulation leads to an optimal approach to gain scheduling. In particular we can interpret gain scheduled designs as approximations to nonlinear overtaking optimal controls. Also these results show that the various linearized point design problems that yield the linear controllers to be scheduled must be related to each other in explicit ways in order for the resulting scheduled controller to be a first-order approximation to an (overtaking) optimal controller for the nonlinear system. Current practice takes no account of these relationships, and the individual point designs are independently formulated to achieve good local performance in a neighborhood of the operating point. In a very intuitive sense, our results show that unless these designs are related in certain ways, local optimality of the individual point designs does not contribute to an overall notion of optimality for the nonlinear controlled system. And of course it is the performance of the nonlinear controlled system that is the end objective.

A related result in [4] deals with a class of infinite-horizon nonlinear optimal control problems. The pseudolinearization transformation is used to investigate a new type of approximate solution for the Hamilton-Jacobi equation and corresponding approximate optimal control law.

Applications of overtaking optimality to optimal tracking problems involving a time-varying linear plant and infinite-horizon quadratic performance index are considered in [5]. In general such problems yield an unbounded performance index for every control and thus must be interpreted as so-called overtaking optimal control problems. A completing-the-square argument is used to derive the overtaking optimal control for a general problem formulation, and various properties are discussed.

In addition to these studies of underlying theoretical frameworks, it became clear that a major limitation of gain scheduling, particularly for the more complex linear controllers that result from modern optimization methods, was the controller interpolation problem. In [7] we propose a method of interpolating linear time-invariant controllers with observer state feedback structure in order to generate a continuously-varying family of controllers that stabilizes a family of linear plants. The interpolation method yields guidelines for the design of gain scheduled controllers. The method is illustrated with the design of a missile autopilot using loop-shaping H-infinity controllers.

Synthesis of gain scheduled controllers often requires that a parameter-varying controller be generated from a finite set of linear time-invariant controllers with more general structure than observer state feedback. Thus in [8] we propose interpolation methods with the property that stability of the linearized closed-loop system is preserved for each fixed value of the scheduling parameter. In addition, slow-variation arguments are presented that establish stability of the nonlinear closed-loop system with gain scheduled controller.

The paper [9] presents an assessment of the state of current research on gain scheduling. Gain scheduling for nonlinear control design is described in terms of general features of the approach and in terms of early examples of applications. Then recent research is discussed, including work on linearization-based scheduling and work on linear parameter varying (LPV) and linear fractional transformation (LFT) approaches.

3. Personnel

Wilson J. Rugh, E.J. Schaefer Professor, Principal Investigator

Hualin Tan, PhD, 1997. *Infinite Horizon Nonlinear Optimal Control and Approximation.* (Now with Rockwell Science Center, Thousand Oaks, CA.)

Daniel J. Stilwell, PhD expected, 1998

4. Journal Publication Activity (1 March 1995 - 28 February 1998)

[1] N. Sureshbabu and W.J. Rugh, "Output Regulation with Derivative Information," *IEEE Transactions on Automatic Control,* Vol. 40, No. 10, pp. 1683 - 1689, 1995.

[2] W.J. Rugh, *Linear System Theory,* expanded second edition, Prentice Hall, Upper Saddle River, NJ, 1996.

[3] N. Sureshbabu and W.J. Rugh, "On Output Regulation for Discrete-Time Nonlinear Systems," *Automatica,* Vol. 33, No. 9, pp. 1683 - 1689, 1997.

[4] H. Tan and W.J. Rugh, "Pseudolinearization and Nonlinear Optimal Control Problems," *IEEE Transactions on Automatic Control,* Vol. 43, No. 3, pp. 386 - 391, 1998.

[5] H. Tan and W.J. Rugh, "On Overtaking Optimal Tracking for Linear Systems," *Systems & Control Letters,* Vol. 33, No. 1, pp. 63 - 72, 1998.

[6] H. Tan and W.J. Rugh, "Nonlinear Overtaking Optimal Control: Sufficiency, Stability, and Approximation," *IEEE Transactions on Automatic Control,* in press, 1998.

[7] D.J. Stilwell and W.J. Rugh, "Interpolation of Observer State Feedback Controllers for Gain Scheduling," *IEEE Transactions on Automatic Control,* in press, 1998.

[8] D.J. Stilwell and W.J. Rugh, "Interpolation Methods for Gain Scheduling," to be submitted, 1998.

[9] W.J. Rugh and J.S. Shamma, "Survey of Research on Gain Scheduling," *Automatica,* invited paper in preparation, 1998.

5. Interactions/Transitions

Preliminary versions of most of the journal publications listed in Section 4 have been presented at professional conferences. Specifically, [3] was presented at the 1995 American Control Conference, and [4] was presented at the 1996 Conference on Information Sciences and Systems. Publication [5] was presented at the 1997 Conference on Information Sciences and Systems, [6] was presented at the 1997 American Control Conference, and [7] was presented at the 1998 Conference on Information Sciences and Systems. Finally, [8] will be presented at the 1998 IEEE Conference on Decision and Control and [9] will be presented as a Plenary Lecture at the 1998 IFAC Nonlinear Control Systems Symposium.

Under the supervision of W.J. Rugh, D.J. Stilwell, a PhD student associated with the

research effort, has been partially supported on a number of flight control projects at the Hopkins' Applied Physics Laboratory. The first project was analysis of an autopilot design for a tactical missile. The missile used nose thrusters, in addition to tail fins, for improved performance. The autopilot was designed by the method of dynamic inversion, and involved gain scheduling. Evaluation of robustness properties of the autopilot was the main activity.

The second project was to analyze potential effects of digital implementation of continuous-time, gain-scheduled autopilot designs on autopilot performance. The two issues addressed were an analysis of hidden coupling terms in discretizing a continuous-time, gain scheduled controller, and the impact of reducing the gain schedule update rate in order to reduce computational burden. Briefly the conclusions were that using a step-invariant discretization did not generate additional coupling terms in the discrete-time autopilot and that update rates could be reduced without affecting performance, with the caveat that assumptions of a slowly-varying setting needed to theoretically guarantee stability become more critical. This activity was part of the STANDARD Missile-2 Block IVA upgrade program for the US Navy.

The final project was involved with gain scheduling on the Advanced Technology Demonstration on "Highly Responsive Missile Control Systems." This interaction focused a portion of our efforts on the problem of controller interpolation that is a central step in the most common applications of gain scheduling to flight control systems. The results of this effort supported the autopilot design competition in the ATD, and has contributed to a design approach that appears promising for the STANDARD Missile-2 Block V program.